MATHEMATICAL PRODIGIES[1]

WHEN scarcely three years old Gauss, according to an anecdote told by himself, followed mentally a calculation of his father's relative in regard to the wages of some workmen, who were to be paid for overtime in proportion to their regular wages, and, detecting a mistake in the amount, he called out "Father, the reckoning is wrong, it makes so much," naming the exact amount. The calculations were repeated and it turned out that the child was correct, while all who witnessed the performance were greatly surprised. He retained an extraordinary ability for mental calculations throughout life and remembered the first few decimals of the logarithms of all numbers, so that he was able to use the data of a logarithmic table in his mental calculations, and hence he possessed a mental slide rule—a unique possession.

Gauss was not only one of the greatest mental calculators on record, but he excelled equally in all branches of pure and applied mathematics. At the age of twenty he discovered the first rigorous proof of the fundamental theorem of algebra, which affirms that every algebraic equation has as many roots as its degree, and at the age of twenty-four he published his great work on the theory of numbers under the title "Disquisitiones Arithmeticæ." Later in life he turned his attention principally to applied mathematics—especially to astronomy and geodesy—and he is generally regarded as the last of the great mathematicians who was preeminent in nearly all branches of mathematical knowledge of his day. He considered mathematics the queen of the sciences and number theory the queen of mathematics.

While Gauss was both a great mental calculator and a great mathematician, and was a real mathematical prodigy, we proceed to consider several who were merely arithmetical prodigies and seemed to have very little general mathematical ability. The greatest of these is Dase, who was born at Hamburg in 1824, and "seems to have been little more than a human calculating machine, able to carry on enormous calculations in his head, but nearly incapable of understanding the principles of mathematics, and of very limited ability outside his chosen field." His extraordinary ability in mental calculation is evidenced by the fact that he was able to multiply mentally two numbers, each of which contained one hundred figures. It took him eight and three quarter hours to perform this feat, which stands in a class by itself, as no other arithmetical prodigy is known to have been able to multiply mentally two numbers each consisting of more than thirty-nine figures. Two forty-figure numbers, Dase was able to multiply mentally in forty minutes, while he would multiply two eight-figure numbers in less than one minute.

What is most surprising about this greatest mental calculator on record is that he was stupid in mathematics. Petersen is said to have tried in vain for six weeks to get the first elements of mathematics into his head, and other eminent mathematicians found that he had very little mathematical ability. Fortunately he was advised by some of the leading mathematicians of his day to turn his extraordinary ability to scientific uses instead of going around the country giving public exhibitions, a career upon which he had entered at the age of fifteen. He calculated many useful tables and was engaged on an extensive factor table at the time of his death. The ease and speed with which he could count the number of books

[1] Read before the Summer School students, University of Illinois, August 5, 1907.

in a case, the number of sheep in a herd, etc., was almost more surprising than his ability as a mental calculator.

Another well-known mental calculator, having even less mathematical ability than Dase, is Buxton, who remained illiterate through life, although his father had some education. He had a wonderful memory for numbers and could call off long numbers from right to left or from left to right with equal facility. On one occasion he squared mentally a thirty-nine-figure number in *two and a half months.* He was extremely slow and in this respect resembled a negro by the name of Tom Fuller who is known as the Virginia calculator. Although entirely illiterate he was able to reduce mentally years and months to seconds and could multiply two nine-figure numbers.

Darboux has called attention to an infant prodigy, of interest both because it relates to a man who afterwards became a very prominent mathematician and also because it is not included in the lists of mathematical prodigies which have recently appeared in this country.[2] Joseph Bertrand was born in Paris in 1822 and was such a delicate child that his parents did not expect him to arrive at manhood, and hence his early education was partly neglected. At the age of four he was sick for a long time and overheard the lessons which were given his brother in the same room. He knew the letters of the alphabet, but nothing more. When he was convalescent his parents brought him a book to look at the pictures, and he relates, in his account of his childhood, that he remembers distinctly how he shocked his parents by reading the text fluently. His frightened father snatched the book from him and commanded that under no pretext should he be allowed to do any work.

The manner in which he learnt elementary algebra and elementary geometry is still more extraordinary. We reproduce his own account:

> At the age of nine I had the great misfortune to lose my father, who, during the last part of his life, resided with my uncle who directed then a school preparing for l'École Polytechnique. The students, the youngest of whom was twice my age, loved me very much and I was very happy in their midst. I was assiduous at their recreations and often followed them to their classes. The teachers regarded me with astonishment but paid little attention to me. The students observed that I understood the work and when a demonstration appeared difficult, the first one who noticed me would run after me, take me up in his arms, and, placing me on a chair so that I could reach the blackboard, made me repeat the demonstration.

At the age of sixteen he entered l'École Polytechnique, and, as the examiner knew that he had already passed the examination for the doctor's degree in science, he gave him some very difficult questions. From one of the answers it appeared that Bertrand had never opened a table of logarithms. The examiner considered this answer an impertinence but gave him the highest grade. At l'École Polytechnique Bertrand says that he was a problem for his companions. He always received the highest grades but he was ignorant of some of the simplest things. For instance, he did not know what words were called adverbs, as he had never prepared a lesson in literature or in science and no teacher had ever asked him to make any calculation of any kind.

Bertrand's extraordinary youth gave rise to many marvelous stories. Fortunately, he wrote a brief account of his early life when he was elected in 1884 to

[2] Scripture, "Arithmetical Prodigies," *American Journal of Psychology,* Vol. IV. (1891), p. 1; Mitchell, "Mathematical Prodigies," *ibid.,* Vol. XVIII. (1907), p. 61. Bertrand was a mathematical prodigy, but he can not be classed among the arithmetical prodigies.

the French Academy. Hence we have a more reliable sketch of this infant prodigy than is possible to obtain in most other cases; for instance, in the case of his countryman, Pascal. The facts that Bertrand was permanent secretary of the Academy of Sciences for more than a quarter of a century, that he is the author of many theorems relating to modern mathematical subjects, and that he lived so recently, add interest to the account of his marvelous early aducation.

In the article already cited, which comes from the Psychological Seminary of Cornell University, Mitchell gives an interesting study of arithmetical prodigies and devotes considerable space to his own case. We add some of his conclusions.[3] "Mathematical precocity, then, stands in a class by itself, as a natural result of the simplicity and isolation of mental arithmetic. There is nothing wonderful or incredible about it. The all-round prodigy like Ampère or Sir William Rowan Hamilton or Macaulay is possible only in a well-to-do and cultured family, where books are at hand and general conditions are favorable, and he must possess genuine mental ability. The musical prodigy, again—Mozart is the stock instance—must come of a musical family, hear music, and have at least some chance to practise, and hence can not long hide his light under a bushel. But the mathematical prodigy requires neither the mental ability and cultured surroundings of the one nor the external aids of the other. He may be an all-round prodigy as were Gauss, Ampère and Safford, but he may also come of the humblest family, and be unable, even under the most favorable conditions, to develop average intelligence."

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[3] Page 39.

SCIENTIFIC BOOKS

THE HARVEY LECTURES FOR 1905–6

This volume consists of thirteen lectures given during the year to the Harvey Society of New York. This organization was founded in the spring of 1905, largely through the initiative of Professor Graham Lusk, for the diffusion of knowledge of the medical sciences by means of lectures given by authoritative research workers. The first volume constitutes a most valuable collection of first-hand information given by some of the most prominent investigators in this country and Europe and the reviewer finds before him an embarrassment of riches from which it is difficult to make a selection since all is good.

The first lecture by Professor Hans Meyer, of Vienna, is devoted to "The Theory of Narcosis" upon which subject no one is more competent to speak than this distinguished exponent of pharmacological research. So soon as scientific medicine began to break away from, or at least to seek, other support than blind empiricism, inquiry into the relation between the physiologic action of a drug and the physical and chemical properties began. One of the first investigations of this kind was carried out by Crum-Brown and Fraser, who discovered that practically all the organic bases in which the pentavalent nitrogen is connected by four of these valences with carbon have the same physiologic action notwithstanding other differences in their constitution and nature. The strong basic properties of these substances seem to be the determining factor in their effects upon the animal cell. Hofmeister and others pointed out that the laxative and diuretic effects of the neutral salts of the alkaline bases are due to their diffusibility and osmotic strength. The anesthetics include many substances that differ from one another chemically, while all depress the central nervous system. Meyer has made a careful study of the distribution coefficient of the narcotics between fatty and watery solutions and arrives at the following explanation of narcosis:

The narcotizing substance enters into a loose physico-chemical combination with the vitally im-